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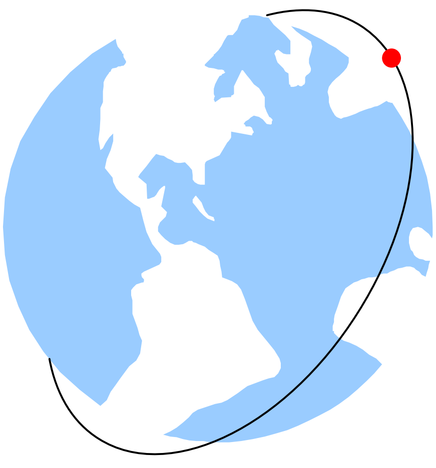
TECHNOTE 2004 - 1

WORKGROUP SATELLITES

Resonant Quadrafilar Helical Antenna



Theory and practice; calculation and measurement of radiation patterns and polarization of an RQHA.

R.W. HOLLANDER

Remarks

This TechNote 2004-1 is an addition to and should be read in conjunction with TechNote 1999-1 and TechNote 2003-1 and describes the recent results of calculations and measurements on the radiation pattern and polarization of an RQHA.

The test results are obtained with an antenna with dimensions described in TechNote 1999-1, p. 12.

TechNotes and the description of how the dimensions of an RQHA, as indicated in the table of TechNote 1999-1, has to be interpreted can be found at www.kunstmanen.nl\antennes as:

RQHA-1999.pdf
RQHA-2003.pdf
RQHA-2004.pdf
RQHA-dims.pdf.

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The RQHA-12 under the Dutch sky

Contents

Abstract

The performance of receiving antennas can be described by the properties of the same antenna while used in transmitting mode. The radiation pattern for emitted waves of a certain polarization is equivalent with the sensitivity pattern for reception for the same polarization. In this TechNote the radiation patterns for Left-handed Circular Polarized (LCP) radiation and Right-handed Circular Polarized (RCP) radiation are described for Resonant Quadrafilar Helical Antennas with the length of one half-loop of a half wavelength.

The effects of helicity (sense of twist) of the helix and the phase difference of the currents are described. A negative helicity results in RCP and a positive helicity in LCP. The phase determines whether the antenna radiates from the top in the upper hemisphere or from the bottom in the lower hemisphere.

Three types are compared: a full turn, half turn and a quarter turn RQHA. The beam width of the dominant polarization gets smaller as more (quarter) turns are used. The axial ratio (a measure of wanted over unwanted polarization) at the horizon is best for a half turn RQHA.

The effect of the ratio of diameter over axial length is also studied. The beam width gets larger and the axial ratio at the horizon gets better with a larger diameter over axial length ratio.

Chapter 1

Maxwell theory for electromagnetic fields

In 1864 James Clerk Maxwell contributed to the understanding of the behaviour of electric and magnetic fields by combining the results obtained by Coulomb, Ampere and Faraday on electric or magnetic fields in a set of equations governing both electric and magnetic effects. This unification resulted in the implication of electro-magnetic (EM) waves, travelling with the speed of light. The Maxwell equations can be presented in several ways. Moreover, depending on the units used, constants like 4π, c (the speed of EM-waves like light in vacuum), ε (the electric permittivity of the medium) or μ (the magnetic permeability of the medium) will show up. In this paper SI-units are used and the Maxwell-equations presented in vector notation are:

$$\begin{aligned} &\nabla \bullet D = \rho \\ &\nabla \times H = J + \frac{\partial D}{\partial t} \\ &\nabla \times E + \frac{\partial B}{\partial t} = 0 \\ &\nabla \bullet B = 0 \end{aligned} \tag{1}$$

The electric displacement $D = \varepsilon E$, with E the electric field strength. The electric charge density is ρ. The magnetic field strength is H. The related magnetic induction $B = \mu H$. The current is denoted by J. The operator ∇ stands for the partial derivatives of the space coordinates, where the combination of $\nabla \bullet$ is the inner product of the ∇-operator and the vector behind the combination (the divergence of the vector) and the combination of ∇ x is the outer product of the ∇-operator and the vector behind the combination (the curl of the vector). In Appendix A some relations for vector calculus are presented.

The first Maxwell equation is the Coulomb equation. The second is the Ampère equation where an extra term $\partial D/\partial t$ is added by Maxwell to account for non-steady currents or charge distributions. The third equation is the Faraday equation. The fourth equation completes the set and is the magnetic equivalent of the first equation; however the magnetic charge density is zero since isolated magnetic monopoles have never been observed.

It may seem that there are too many observables in (1), however in general ε and μ are not scalars but 3x3 matrices. In homogeneous media these matrices reduce to scalars and in vacuum become respectively ε_o and μ_o:

$$\begin{aligned} &\varepsilon_0 = 0{,}886\, pF/m \\ &\mu_0 = 4\pi.10^7\, Henry/m \\ &c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 299{,}7.10^3\, km/s \end{aligned} \tag{2}$$

From now on vacuum conditions are assumed for normal air. We first look at the third and fourth Maxwell equation. From $\nabla \bullet B = 0$ it is possible to consider B as originated by a vector potential A:

$$B = \nabla \times A \tag{3}$$

It is a general property of vector fields that the divergence of the curl of a vector is zero (App.A).

With (3) the third Maxwell equation can be written as:

$$\nabla \times \left(E + \frac{\partial A}{\partial t} \right) = 0 \tag{4}$$

Since the curl of the gradient of a scalar is always zero it is possible to think of the term between brackets as a gradient of a scalar potential Φ:

$$E + \frac{\partial A}{\partial t} = -\nabla \Phi \tag{5}$$

Using the third and fourth Maxwell equation, the homogeneous equations, and general rules for vector fields it is thus possible to introduce a vector potential A and a scalar potential Φ as generators for B and E. Introducing these potentials in the first and second Maxwell equation, the inhomogeneous equations, we get for the first:

$$\begin{gathered} \nabla \bullet E = \frac{\rho}{\varepsilon_0} = \nabla \bullet \left(-\nabla \Phi - \frac{\partial A}{\partial t} \right) \quad \text{or} \\ \nabla^2 \Phi + \frac{\partial}{\partial t} (\nabla \bullet A) = -\frac{\rho}{\varepsilon_0} \end{gathered} \tag{6}$$

and for the second:

$$\begin{gathered} \nabla \times \frac{\nabla \times A}{\mu_0} = J - \varepsilon_0 \left(\nabla \frac{\partial \Phi}{\partial t} + \frac{\partial^2 A}{\partial t^2} \right) \quad \text{or} \\ \nabla^2 A - \varepsilon_0 \mu_0 \frac{\partial^2 A}{\partial t^2} - \nabla \left(\nabla \bullet A + \varepsilon_0 \mu_0 \frac{\partial \Phi}{\partial t} \right) = -\mu_0 J \end{gathered} \tag{7}$$

The four Maxwell (first order differential) equations are now reduced to only two (second order differential) equations involving the generating potentials A and Φ. The equations (6) and (7) are however coupled. How to proceed depends on the kind of problem that has to be solved.

The potentials A and Φ are not unique. When we add to A a gradient of a scalar Θ:

$$A \to A + \nabla \Theta \tag{8}$$

we get from (3):

$$B = \nabla \times (A + \nabla \Theta) = \nabla \times A + \nabla \times (\nabla \Theta) = \nabla \times A \tag{9}$$

leaving B unchanged. In order to leave E also unchanged we have to combine the change in A with a change in Φ:

$$\Phi \to \Phi - \frac{\partial \Theta}{\partial t} \tag{10}$$

we get from (5):

$$E = -\frac{\partial(A + \nabla\Theta)}{\partial t} - \nabla\left(\Phi - \frac{\partial\Theta}{\partial t}\right) = -\frac{\partial A}{\partial t} - \nabla\Phi \qquad (11)$$

This means that we are free to change A as long as we also make an appropriate change in Φ. The relation between A and Φ that we may chose and that is useful in the description of many problems is:

$$\nabla \bullet A + \varepsilon_0 \mu_0 \frac{\partial\Phi}{\partial t} = 0 \qquad (12)$$

then the two coupled Maxwell equations (6) and (7) become two independent 'wave' equations, one for A and one for Φ:

$$\begin{aligned} \nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} &= -\frac{\rho}{\varepsilon_0} \\ \nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} &= -\mu_0 J \end{aligned} \qquad (13)$$

The scalar potential Φ is constructed from the charge density ρ and the vector potential is constructed from the current J.

Another choice, which is useful in the description of radiating systems like antennas, is simply:

$$\nabla \bullet A = 0 \qquad (14)$$

This is called the 'Coulomb', 'radiation' or 'transversal' gauge. Then we get from (6):

$$\nabla^2\Phi = -\frac{\rho}{\varepsilon_0} \qquad (15)$$

Now the scalar potential Φ is the instantaneous Coulomb potential due to the charge density ρ. It is now instructive to consider the current J as the vector sum of two perpendicular currents J_L and J_T. When we take J_L in the direction of r and J_T perpendicular to r in the (θ,φ)-plane (using spherical coordinates (r,θ,φ)):

$$\begin{aligned} J_L &= (\hat{u}_r \bullet J)\hat{u}_r \\ J_T &= -\hat{u}_r \times (\hat{u}_r \times J) = (\hat{u}_\theta \bullet J)\hat{u}_\theta + (\hat{u}_\varphi \bullet J)\hat{u}_\varphi \end{aligned} \qquad (16)$$

where $\hat{u}_r$, $\hat{u}_\theta$, and $\hat{u}_\varphi$ are the unit vectors in the respective directions. It follows that [1]:

$$\nabla\frac{\partial\Phi}{\partial t} = \frac{J_L}{\varepsilon_0} \qquad (17)$$

and equation (7) becomes the 'wave' equation for the vector potential A:

$$\nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\mu_0 J_T \qquad (18)$$

From (18) it is clear that A is generated by the 'transversal' current only!

In the common situation that the charge density ρ is zero also the generated scalar potential (and its gradient) is zero. Then we can easily find E from A (11), which is generated by the transversal current according (18):

$$E = -\frac{\partial A}{\partial t} \tag{19}$$

When the currents are periodic sinusoidal in time we can write:

$$J_T = J_{T,A} \exp(j\,(\omega t + \delta)) \tag{20}$$

where $J_{T,A}$ is the amplitude of the transversal current, ω is the angular frequency (equal to $2\pi f$) and δ is an arbitrary phase angle. The potential A and thus the field E at point $P(r)$ some distance $|r\text{-}r_s|$ from the current element at r_s is generated at a later time $t+|r\text{-}r_s|/c$ due to the finite speed of light. The potential $A(r,t)$ is thus made up of contributions from currents in elements located at r_s at the time $t\text{-}|r\text{-}r_s|/c$:

$$\begin{aligned} J_T &= J_{T,A} \exp\left[j \left\{ \omega \left(t - \frac{|r - r_s|}{c} \right) + \delta \right\} \right] \\ J_T &= J_{T,A} \exp(j\,\omega t) \exp(j\,\delta) \exp(-jk|r - r_s|) \end{aligned} \tag{21}$$

The time dependence of A will be consequently also $\exp(j\omega t)$ with a phase factor $\exp(j\delta)$ of the current elements and hence the partial derivative to t in (19) will yield a factor $j\omega$ and the second derivative in (18) a factor $-\omega^2$.

If we restrict ourselves to one frequency we can drop the common factor $\exp(j\omega t)\exp(j\delta)$ and write (18) and (19) as (in fact we should perform a Fourier transform from the time domain to the frequency domain):

$$\begin{aligned} \nabla^2 A + \frac{\omega^2}{c^2} A &= (\nabla^2 + k^2)A = -\mu_0 J_{T,A} \\ E &= -j\omega\, A \end{aligned} \tag{22}$$

where k is the wave number defined by:

$$k = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda} \tag{23}$$

It should be noted that the definition of the current as in (21) has consequences for further definitions of phases and the sense of circular polarization. In (21) the choice is made for time going in the positive sense as $\exp(+j\omega t)$. This means that an outgoing wave has the phase $\exp(-j\omega r)$! When comparing formulas in this paper with formulas in other publications one should always first look at the definition in the other paper like (21)!

Chapter 2

The E-field of current elements

The problem is now to solve the inhomogeneous differential equation (22) for A to find the potential A at the position $P(r)$ indicated by the vector $r(r,\theta,\varphi\ ),$ from the given current amplitude $J_{T,A}$ at r_s. It happens that, the solution is quite simple:

$$A(r) = \mu_0 \int J_{T,A}(r_s) \frac{\exp(-jk\,|r - r_s|)}{|r - r_s|} dr_s \qquad (24)$$

where the integration involves the volume in which $J_{T,A}(r_s)$ is not zero (the volume where the antenna is located). In the 'far' field we can approximate the distance between a current element s at r_s and the position $P(r)$ in space where we calculate the potential, as it shows up in the denominator, by the distance between the centre of the antenna and $P(r)$, which is simply the length of r when we take the origin of our coordinates at the centre of our antenna. This can be done as long as $r >> r_s$. The far field approximation is valid for distances larger than 2.5 times the wavelength (when looking at an antenna with dimensions of ~0.5 wavelength).

The same term $|r - r_s|$ in the exponential has to be treated differently, since here a phase is involved. In the far-field approximation the vector from the current element to $P(r)$ is parallel with the vector from the origin to $P(r)$. Here we can take:

$$|r - r_s| = |r| - \frac{r \bullet r_s}{|r|} = |r| - \hat{u}_r \bullet r_s \qquad (25)$$

Using these approximations for the far field situation and changing the integration over all r_s in the volume to the variable s, measured along the current elements with the direction of these elements denoted by the unit vector $\hat{u}_s$, we get:

$$\begin{aligned} E(r) &= -j\omega\mu_0 \frac{\exp(-jkr)}{4\pi r} \int J_{T,A}(s)\ \exp(jk\hat{r}_s \bullet \hat{u}_r) ds \\ &= -j\omega\mu_0 \frac{\exp(-jkr)}{4\pi r} \int -\hat{u}_r \times (\hat{u}_r \times \hat{u}_s) J_A(s)\ \exp(jk\hat{r}_s \bullet \hat{u}_r) ds \end{aligned} \qquad (26)$$

The vectors $\hat{u}_\theta$, $\hat{u}_\varphi$, $\hat{u}_s$, $\hat{u}_r$ and $\hat{r}_s$ are respectively:

- the unit vector in the θ-direction,
- the unit vector in the φ-direction,
- the unit vector in the direction of the current at the position s,
- the unit vector in the r-direction,
- the vector from the origin to the current element at s.

The term $\hat{u}_r$ x $(\hat{u}_r$ x $\hat{u}_s)$ makes that only the component of J perpendicular to r (transversal) is used. This produces fields in the θ and φ directions only and not in the r direction! With the vector equality:

$$-\hat{u}_r \times (\hat{u}_r \times \hat{u}_s) = +(\hat{u}_\varphi \bullet \hat{u}_s)\,\hat{u}_\varphi + (\hat{u}_\theta \bullet \hat{u}_s)\,\hat{u}_\theta \qquad (27)$$

for the usual definition of the spherical coordinates we can write for the components, including the phase factor exp($j\delta$) (the factor exp($j\omega t$), common to all currents, potentials and fields is omitted):

$$
\begin{aligned}
E_\theta(r) &= -j\omega\mu_0 \exp(j\delta)\frac{\exp(-jkr)}{4\pi r}\int J_A(s)\,\hat{u}_s \bullet \hat{u}_\theta \exp(jk\hat{r}_s \bullet \hat{u}_r)\,ds \\
E_\varphi(r) &= -j\omega\mu_0 \exp(j\delta)\frac{\exp(-jkr)}{4\pi r}\int J_A(s)\,\hat{u}_s \bullet \hat{u}_\varphi \exp(jk\hat{r}_s \bullet \hat{u}_r)\,ds
\end{aligned}
\qquad (28)
$$

where the total field is the vector sum of the θ and φ components:

$$
E(r) = E_\theta(r)\hat{u}_\theta + E_\varphi(r)\hat{u}_\varphi \qquad (29)
$$

The r component of the E-field is zero (EM-fields are transversal). The term exp($j\delta$) is the relative phase of the current in the half loop we are looking at (for RCP we need a feed in quadrature; $\delta_{1,3}$ = 0, $\delta_{2,4}$ = -π/2 which makes exp($j\delta$) = 1, resp. -j).

Any field can be decomposed into orthogonal fields. The E-field with the two orthogonal components E_θ and E_φ can be written as two linearly polarized eg. E_H (horizontal) and E_V (vertical), or two circularly polarized fields E_R (right handed) and E_L (left handed). For the emitted circular polarization in the positive r-direction we can write:

$$
\begin{aligned}
\hat{E} &= E_R\hat{u}_R + E_L\hat{u}_L \\
\hat{u}_R &= \frac{1}{\sqrt{2}}\left(\hat{u}_\theta - j\,\hat{u}_\varphi\right) \\
\hat{u}_L &= \frac{1}{\sqrt{2}}\left(\hat{u}_\theta + j\,\hat{u}_\varphi\right)
\end{aligned}
\qquad (30)
$$

The new base vectors $\hat{u}_R$ and $\hat{u}_L$ are ortho-normal:

$$
\begin{aligned}
\hat{u}_R \bullet \hat{u}'_L &= \hat{u}_L \bullet \hat{u}'_R = 0 \\
\hat{u}_R \bullet \hat{u}'_R &= \hat{u}_L \bullet \hat{u}'_L = 1
\end{aligned}
\qquad (31)
$$

We have to realize that the definition for the sense of polarisation is: the direction of rotation of the E-vector looking at the oncoming wave [1]. This in not the same as the definition of helicity, which is the sense of rotation in the propagation direction of the wave! An RCP-field has negative helicty! From (29) and (30) we get:

$$
\begin{aligned}
E_R &= \frac{1}{\sqrt{2}}\left(E_\theta + j\,E_\varphi\right) \\
E_L &= \frac{1}{\sqrt{2}}\left(E_\theta - j\,E_\varphi\right)
\end{aligned}
\qquad (32)
$$

The signs in (30) and (32) are sensitive to the definition in (21)!

The axial ratio, a measure of circular polarization, can be defined in several ways. If we use:

$$
AR = \frac{\left\|E_R\right| - \left|E_L\right\|}{\left|E_R\right| + \left|E_L\right|} \qquad (33)
$$

then we have pure circular polarization for AR = 1 and pure linear polarization for AR = 0.

Chapter 3

The E-field of an RQHA antenna

The radiation pattern of an antenna is equivalent with the sensitivity pattern of the same antenna used in receiving mode. Since we want to know the sensitivity for RCP-waves from APT-transmissions we have to calculate E_R as a function of (θ,φ). Moreover, it is interesting to know the sensitivity for LCP-waves as well, since reflected RCP-waves reverse polarization and become LCP. The interference of direct and reflected waves causes noise-bands in the images, predominantly when satellites are near the horizon. However, when our antenna is insensitive for LCP our images will not suffer from these noise-bands.

We thus obtain the wanted pattern by calculating E_R and E_L (32). This was first done for an RQHA by Kilgus [2]. He presented formulas for the φ-components of the radial and helical parts of an RQHA. He used some approximations to make the problem solvable for him (in 1969). Wang [3] has calculated the fields of quadrafilar antennas using the formulas of Kornhauser [4] for fields of helical wires. Nakano et. al. [5] have presented a paper about helical antenna analysis which was helpful for the development of the now presented program. With new and much more powerful programs like Matlab, integrals in vector-notation can easily be solved. By comparing the present vector equations with the formulas of Kilgus however we run into troubles. Kilgus described the radial and helical parts, using phases and current directions in a very clever way to get a simple set of formulas. Now we want to calculate all field-components and E_R, E_L and AR, without approximations except for the assumption of the current distribution. So we have to describe the problem in a much more general way. We will take the integrals over the current elements starting at one point along a full loop (the distal point was chosen as in Kilgus article), consisting of two half loops. The four half loops are all of the length of a half wavelength. The phase along the loops runs from zero to 2π when we are back at the distal point after one full loop. We assume a cosine distribution:

$$J_A(s) = J_0 \cos(ks) \qquad (34)$$

with maxima at the distal-point at the bottom ($J_A(0) = J_A(\lambda) = +J_0$) and at the feed-point at the top ($J_A(\lambda/2) = -J_0$). The program can calculate fields for full-turn, half-turn and quarter-turn quadrafilar antennas. The free design parameters are:

- frequency
- number of turns (1, ½ or ¼)
- ratio of diameter over axial length
- the distance from the centre of the RQHA to the point where the field strength is calculated.
- the helicity of the helix
- the phase of half loops 2,4 relative to half loops 1,3
- the azimuth angle for the elevation scan perforformed

The fields are calculated as a scan over all angles θ for the selected angle φ. Fields of an RQHA are independent of φ, as was stated by Kilgus and is once more verified by these calculations. The relevant polar plots of E_θ, E_φ, E_R, E_L are shown in Fig.1.

Although it is simple to calculate the absolute field strength at a selected distance (as long as the far field approximations are valid), this is not done. Only the angular dependence is shown as this is the relevant information for radiation patterns. The factor:

$$factor = \omega\mu_0 J_0 \frac{\exp(-jkr)}{4\pi r} \qquad (35)$$

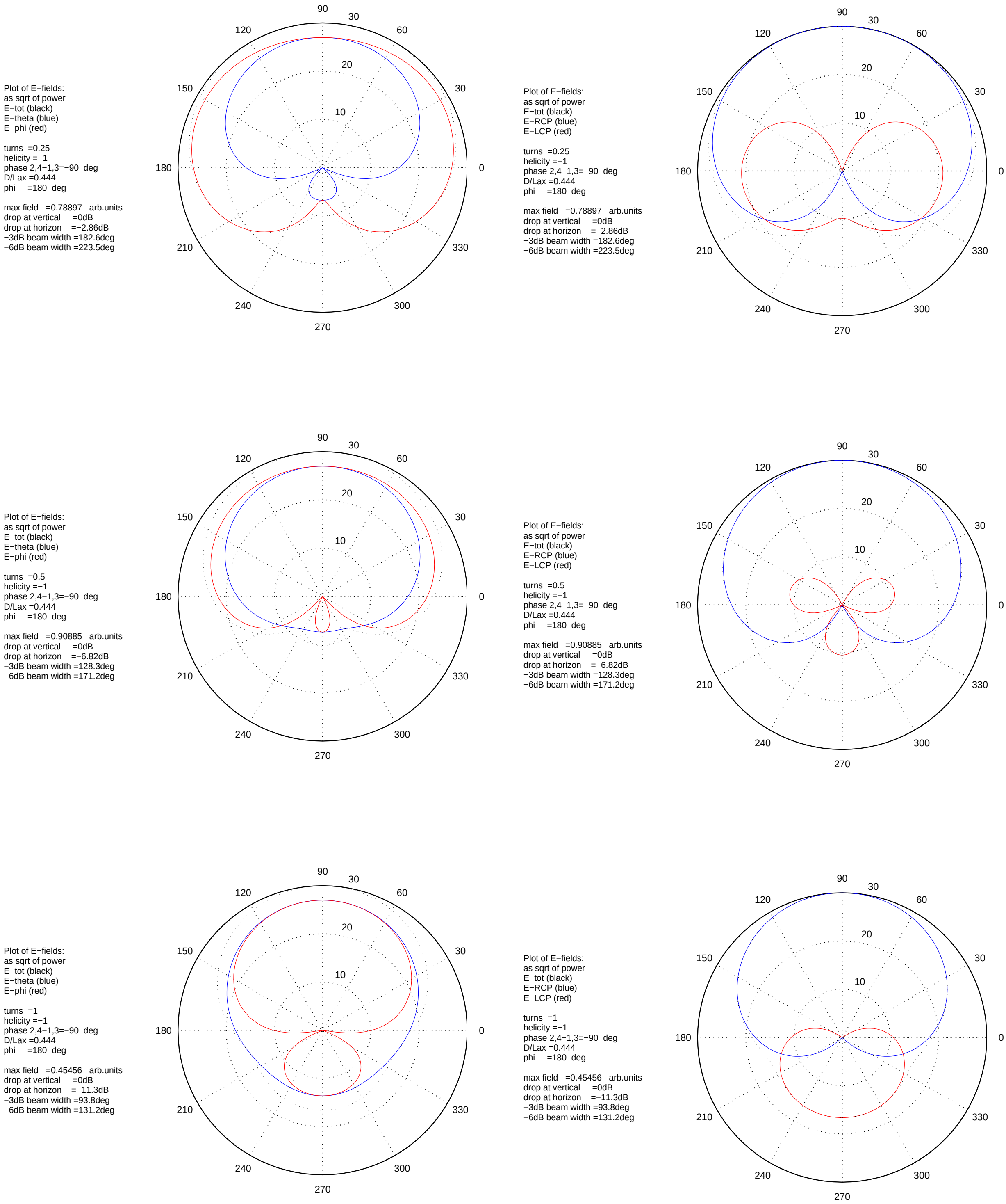


Fig.1 Radiation patterns of an RQHA with D/L of 0.444 as a function of the number of turns.

is ignored. This makes the choice of the distance and frequency as design parameters a bit strange since the presented results do not depend on these parameters. However, these parameters are incorporated in the program so that when not only the radiation pattern but also the absolute field strength is wanted the program is able to calculate this simply by multiplication of the results with the above mentioned factor.

In the calculations the origin is taken in the centre of the antenna (Kilgus takes the distal point as origin). The half loops are numbered 1,2,3,4 counting CCW looking at the top downwards (Kilgus counts 1,3,2,4). The phase δ of the current in the half loops 2 and 4 (inductive large loop) has to be $-\pi/2$ relative to the half loops 1 and 3 (capacitive small loop) to get RCP in the upper hemisphere. Note that the twist of an RQHA for RCP-APT reception (helicity of helix=-1) is opposite as in the LCP-example in Kilgus article [2] (helicity=+1)!

The code of the program is supplied in Appendix B. The definition of the geometry is given in Fig.B1. With the program it will be easy to tailor the ratio of diameter over axial length to the desired radiation pattern. In Table-I the angle θ for maximal sensitivity (0dB), the sensitivity drop (in dB) looking up vertically and the sensitivity drop at the horizon are shown as well as the -3dB and -6dB beam width.

Table-I, Simulation results, pattern characteristics as a function of D/L

D/L	max @ θ degrees	up/max dB	hor/max dB	-3dB beam width	-6dB beam width
0.200	0	0	-5.0	154.2	190.9
0.314	0	0	-6.0	139.4	179.7
0.369	0	0	-6.4	134.0	175.7
0.444	0	0	-6.8	128.3	171.2
0.612	0	0	-7.3	120.2	164.9
0.840	0	0	-7.6	115.0	161.2

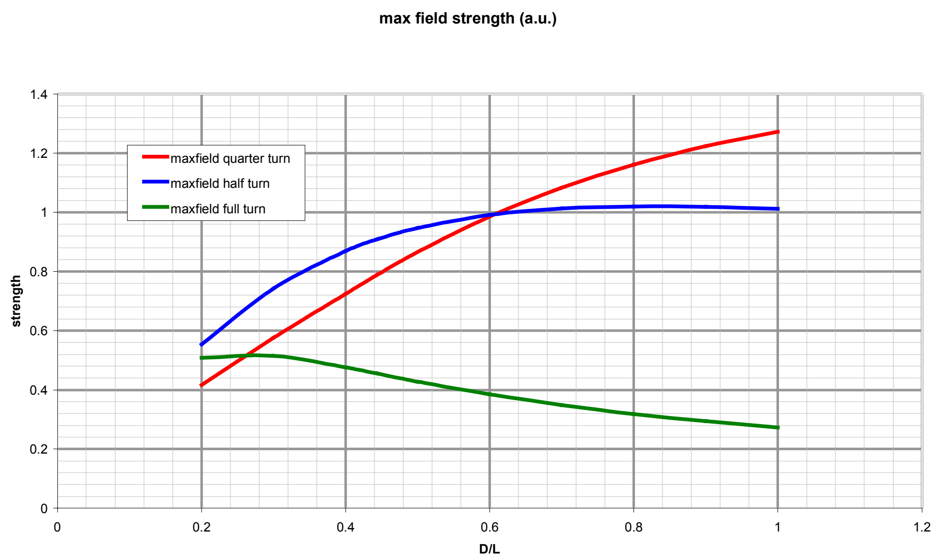


Fig. 2a Maximum field strength as a function of D/L in arbitrary units.

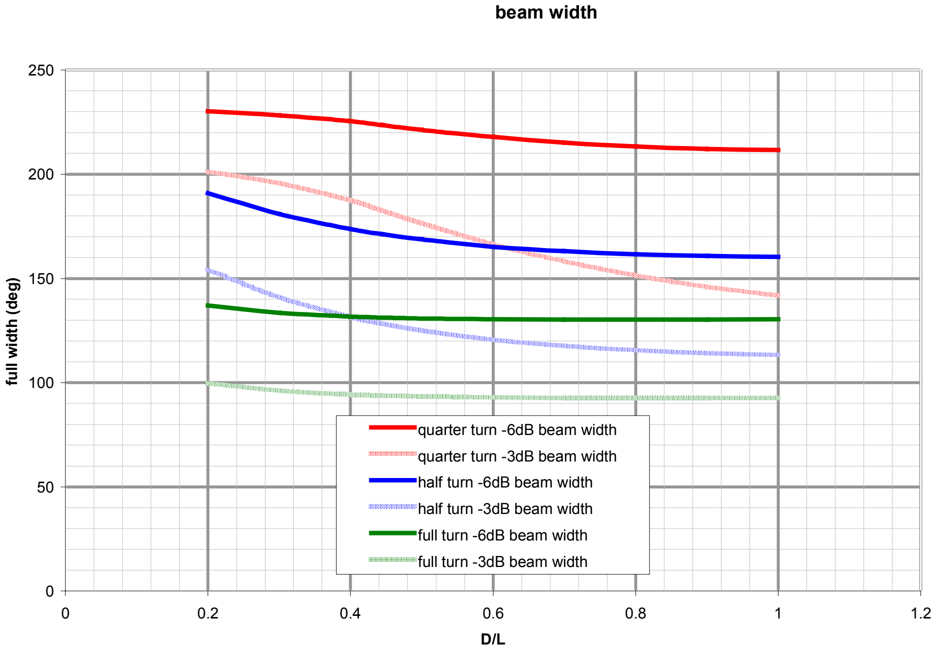


Fig. 2b Beam width (3dB and 6dB) as a function of D/ L (# of turns as parameter).

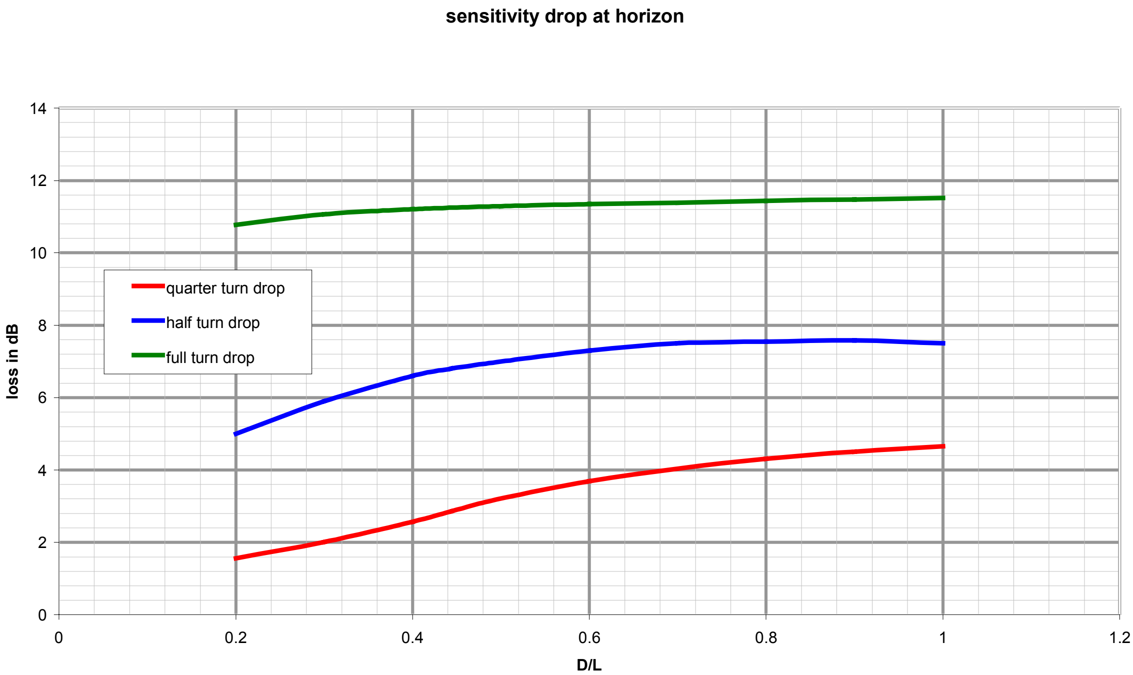


Fig. 2c Sensitivity drop at the horizon as a function of D/L (# of turns as parameter).

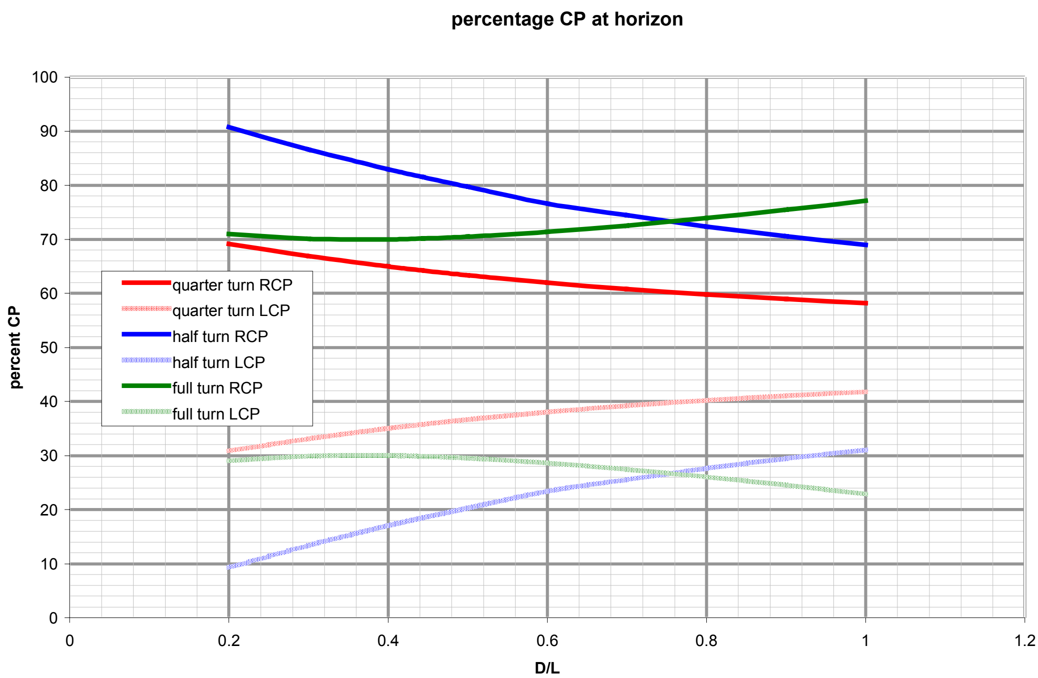


Fig. 2d Percentage of circular polarisation at the horizon as a function of D/L (# of turns as parameter).

Chapter 4

Comparison of calculated and measured radiation patterns and polarization

The most interesting test for the pattern of an RQHA is to measure $E_R(\theta,\varphi)$ and $E_L(\theta,\varphi)$ and to compare the obtained results with the calculated ones. For such a test we need a pure RCP transmitting antenna and an identical LCP one. This is not simple. The best choice is to take two dipole antennas, one a quarter wavelength behind the other at 90 degrees fed in phase as indicated in Fig.3. To avoid different scattering conditions in the RCP and LCP set-up use a 45 degrees geometry with the horizon.

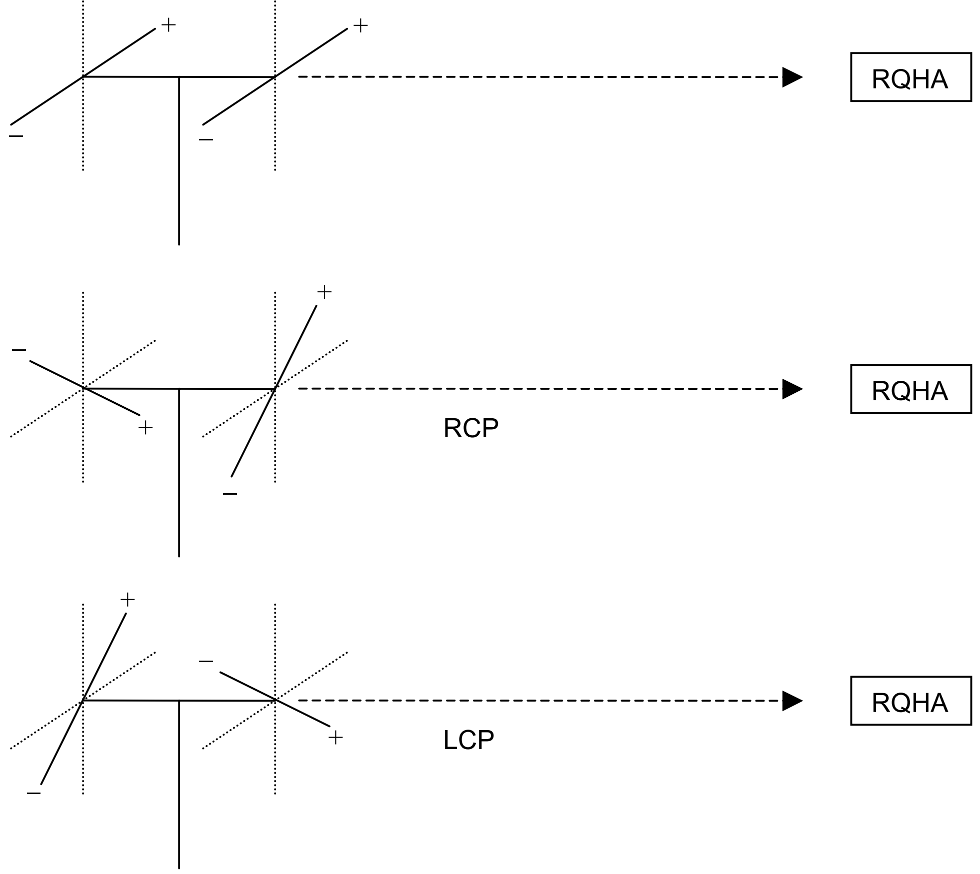
A θ scan (or EL scan) is made by tilting the RQHA at a selected angle φ (AZ fixed) in two steps: first measure the signal strength with θ ranging from 0 to 180 degrees, than rotate φ by 180 degrees and complete the full scan by making the first step in reverse order, which is then equivalent with θ from 180 to 0 degrees. Make such a full scan with the transmitting antenna in RCP and LCP geometry as well.

In fact we have to repeat this procedure for all values of the azimuth angle φ, at least for two, preferably 90 degrees apart. An RQHA should be insensitive to φ, but this has to be checked. If one wants to be sure φ scans can be made at different θ, at least two, preferably at θ = 45 and 90 degrees (EL=45 and 0 degrees resp.).

Another way to get an impression of the circular polarization properties of an antenna is to measure $E_\theta(\theta,\varphi)$ and $E_\varphi(\theta,\varphi)$ with a linear polarized transmitting antenna; however we can not get information whether the antenna is RCP or LCP. From (32) and (33) we see that if we find $E_\theta = E_\varphi$ at all angles (θ,φ) then the antenna is pure circularly polarized (RCP or LCP). With the scan method described above we need a vertically polarized transmitting antenna to measure E_θ and for E_φ a horizontally polarized antenna. A simple way to measure $E_\theta(\theta,\varphi)$ and $E_\varphi(\theta,\varphi)$ in one scan is to use a spinning dipole as transmitting antenna. Again θ scans has to be made at least at two azimuth angles φ, preferably 90 degrees apart.

For the measurement of the gain we need a (calibrated) reference antenna, eg. a dipole antenna. Using circular polarized fields, the signal from a dipole, which senses only E_θ or E_φ, will be 3 dB less than from an ideal RQHA. On the other hand, using linear polarized fields, the signal from an ideal RQHA is 3 dB less than from the reference dipole lined up with the transmitting antenna. The impedance of all antennas has to be the same, e.g. 50Ω.

Fig.3. Geometry for measurement of circular polarization.



Measurement set-up

Recently a little group was formed from members of the Dutch Workgroup Satellites or as we call it the ‘Werkgroep Kunstmanen’, who together did the measurements: Harrie van Deursen, Arne van Belle, Peter Smits and myself, Robert Hollander. Two members of this group can be seen in action in Fig.6.

A mobile, battery operated, set-up was made by the group partly from existing units, while also some new units had to be made. A small transmitter and impedance matching network feed the transmitting antenna, which consists of two dipoles as indicated in Fig.3, to produce circular polarized fields. An attenuator is used to adjust the power, so that the receiver at approximately 100 yards is not overloaded. About 1 μW was used in the experiments. The AUT (antenna under test) was the RHQH-12, connected with a 50Ω cable with an electrical length of ¾ lambda to an antenna amplifier (HRA-137). This ¾ lambda is the total length of the cable from the top of the antenna to the amplifier, thus including the 'internal' cable of the 'infinite BALUN' and the 'length' of the connectors. This cable transforms the real impedance of the antenna at its terminals at the top of 31.5Ω @ 137.5 MHz to a real impedance of 79Ω, suitable for the antenna amplifier input.

The receiver was a HRX-137, fitted with a special buffered signal strength output. The output voltage as a function of the input voltage was calibrated. Although the output is approximately a log-function of the input, several wiggles show up in the curve due to the internal circuit of the CA3089. There are better signal monitor IC's, but a 9-th order polynomial fit represented our calibration curve nicely, so that this fitted curve could be used in the analysis of the measurements.

The output voltage was measured with an 8-input ADC-unit, made from a kit from Conrad-Elektronika. This ADC-unit is controlled by the control lines DTR and RTS from a serial port of a laptop and read out by the status lines DSR and CTS. A measuring program has been written to collect the data and store the data on disk. At intervals of selectable length, e.g. one sec, a selectable number of samples, e.g. 10, are converted and averaged to suppress noise to less than 1 mV. Not only the signal strength is recorded, also the voltages representing the angles of the AZ (azimuth) and EL (elevation) rotors, produced with multi-turn linear pot-meters coupled to the rotor shafts, are sampled almost at the same moment as the signal voltage. This procedure produces three values per sample record, completely defining the required information, independent of the speed of the rotors and without synchronisation problems. The relations between readings and angles were calibrated, and proved to be very linear (high impedance loading of the pot-meters was used).

A 12V battery from which a stabilised 15VDC is made by a DC-DC converter, specially made for this occasion, powers the whole set-up. It is the supply for the laptop, the receiver, the ADC-unit and the rotor steering. The receiver has an additional internal 12V stabiliser and the ADC-unit a 9V stabiliser, a second 5V stabiliser for the pot-meters and a 5V reference for the ADC. Recordings are +/- 1mV on a range of 5V.

Measurements

For our experiment we choose a site with two hills about 100 yards apart, one for the transmitter and one for the AUT. First we tested the effect of the circular polarisation of the transmitting antenna with a receiving antenna consisting of a simple dipole. A variation of 5.6 dB was found by an AZ-scan when the antennas were in line. The use of a reflector behind the receiving dipole reduced this variation to 2.8 dB. We concluded, that the reception of the dipole is seriously influenced by the material at the backside of the antenna, in our case a rather massive piece of metal from the rotors. At the transmitting side a crossed dipole without reflectors was used. There we did not have any metal in the vicinity, except for the feed cable. We did not realise at that time that ground reflections are the main cause of this difference between horizontal and vertical field strengths. The circular polarisation is not perfect, but good enough for the measurement of the radiation pattern. For a cross-polarisation measurement, to be done yet, we have to use pure LCP and RCP fields.

Analysis

The dB values in Fig. 4 (vertical scale) are relative to 1μV at the input of the receiver from which the gain of the antenna amplifier should be subtracted to get the antenna signal. However, we are interested in the antenna gain and to get the antenna gain it is better to make a comparison with a dipole antenna, from which the gain is well defined. In our experiments this time we changed the transmitting power between the dipole and the RQHA measurements, as we realised after the measurements, so this comparison cannot be made this time. Only the patterns were measured, as shown in Fig.4.

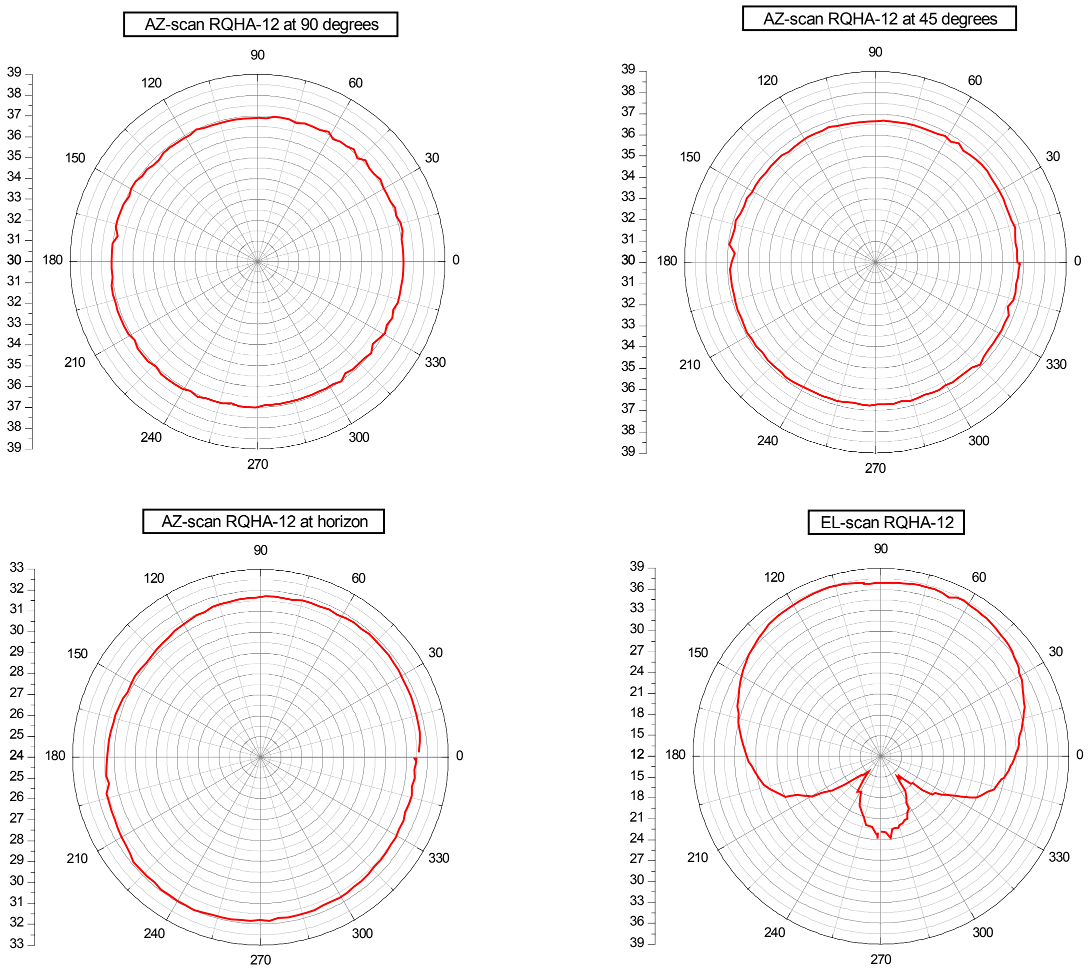


Fig. 4 Measured radiation patterns of an RQHA-12 with a D/L of 0.444. Three AZ-scans at different elevation angles show excellent constant sensitivity in all AZ-directions.

The absolute constant sensitivity at all AZ-angles is astonishing, although it was meant to be so. Also the EL-scan is very symmetric and close to what it should be (see Fig.5). The ratio of diameter over axial length of 0.44 was selected according to Wang [3] to get a decrease of 6dB at the horizon (remember, I live in the city and wanted some reduced sensitivity at the horizon to avoid terrestrial interference) and we indeed find a reduction of -6dB. Even the lobe at the bottom, which should not be there, is as was also found by Kilgus [2] (see Fig.5), the inventor of the quadrafilar antenna, on his antennas 30 years ago.

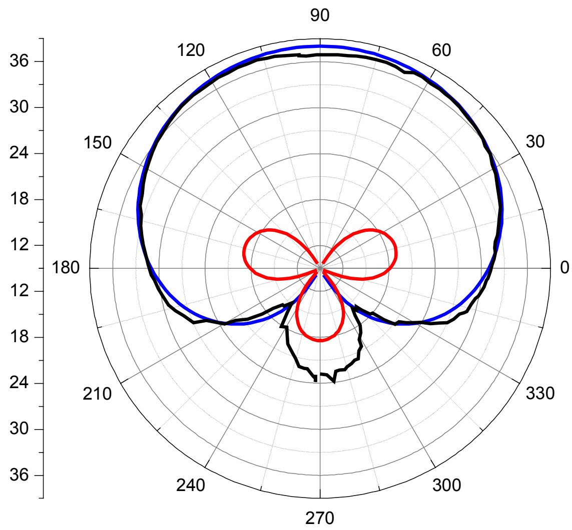


Fig. 5 Comparison of the measured radiation pattern of an RQHA-12 with a D/L of 0.444 with theory. The blue curve is the pattern for RCP and the red curve is for LCP.

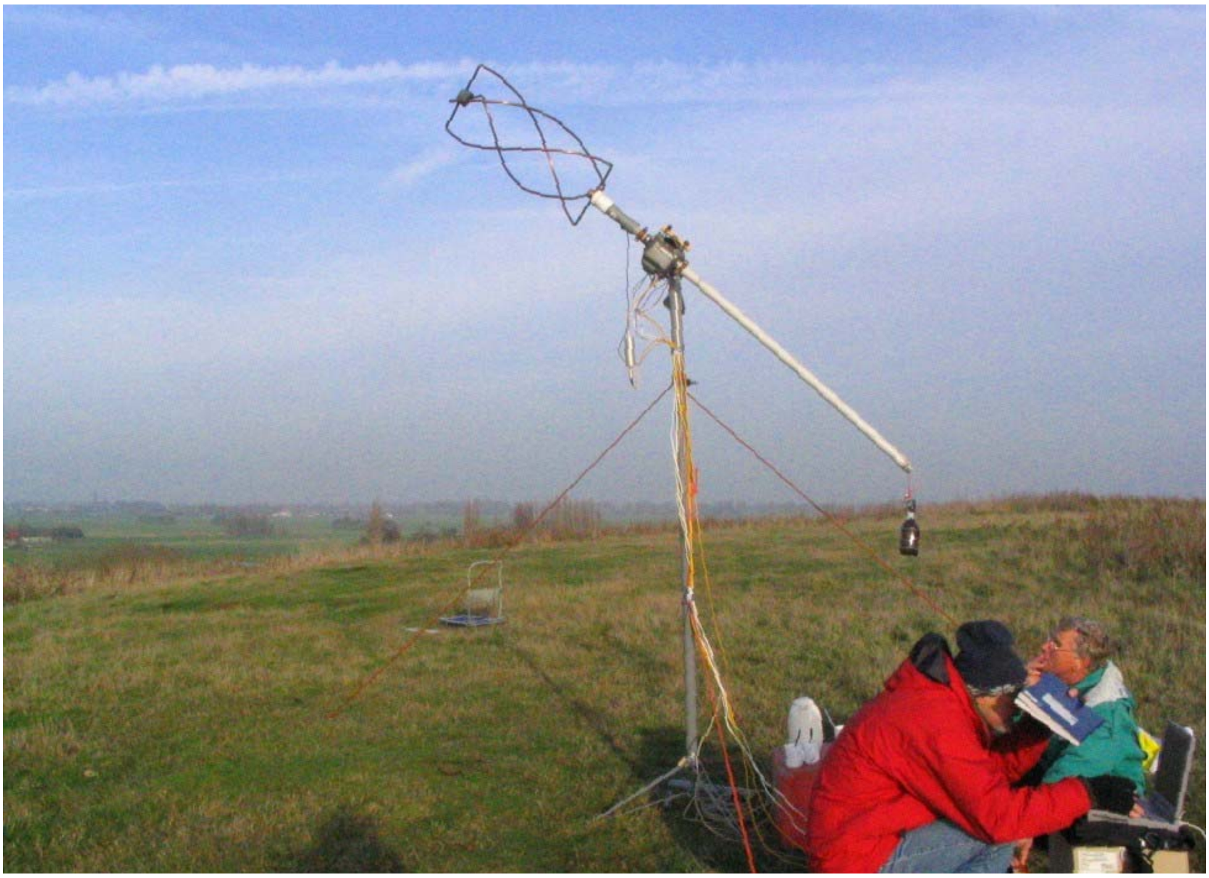


Fig. 6. The RQHA-12 under test (notice the weight to balance the set-up)

Conclusion

Although expectations were high, we are astonished by the results. It seems that all the care in making the construction pays off. From the electrical measurements it was already clear, that the phasing of the RQHA-12, with an inductive and a capacitive loop, is almost perfect. Now it is also clear that the radiation pattern is close to perfect. We conclude that the RQHA is not only theoretically an ideal antenna for the reception of polar orbiting satellites but that the RQHA can also be an ideal antenna in practice, when properly made.

References

[1] J.D. Jackson, Classical Electrodynamics, 2nd ed. John Wiley & Sons, 1975.

[2] C.C. Kilgus, Resonant Quadrafilar Helix, IEEE Transactions on Antennas and Propagation, Vol. **AP-17**, May 1969, pp 349-351.

[3] H.S.C. Wang, Theoretical design and predictions of volute antenna performance, IEEE Transactions on Antennas and Propagation, Vol. **AP-39**, No.8, August 1991, pp 1227-1230.

[4] E.T Kornhauser, Radiation field of helical antennas with sinusoidal current, Journal of Applied Physics, Vol.**22**, No.7, July 1951, pp 887-891.

[5] H. Nakano et al. Helical Antenna analysis using the Method of Moments and the Finite-Difference Time Domain Method, ECTI Transactions on Electrical eng. Electronics and Communications, Vol.**1**, No.1, August 2003, pp17-26.

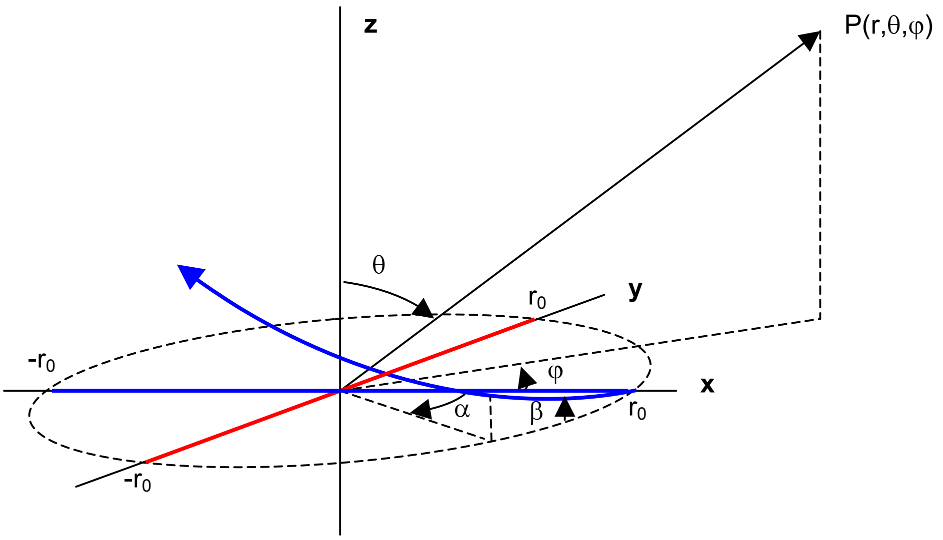
Appendix A Vector calculus

a, b and c are vectors, ψ is a scalar.

$$a \bullet (b \times c) = b \bullet (c \times a) = c \bullet (a \times b)$$
$$a \times (b \times c) = (a \bullet c)b - (a \bullet b)c$$
$$(a \times b) \bullet (c \times d) = (a \bullet c)(b \bullet d) - (a \bullet d)(b \bullet c)$$
$$\nabla \times \nabla \psi = 0$$
$$\nabla \bullet (\nabla \times a) = 0$$
$$\nabla \times (\nabla \times a) = \nabla(\nabla \bullet a) - \nabla^2 a$$
$$\nabla \bullet (\psi a) = a \bullet \nabla \psi + \psi \nabla \bullet a$$
$$\nabla \times (\psi a) = \nabla \psi \times a + \psi \nabla \times a$$
$$\nabla(a \bullet b) = (a \bullet \nabla)b + (b \bullet \nabla)a + a \times (\nabla \times b) + b \times (\nabla \times a)$$
$$\nabla \bullet (a \times b) = b \bullet (\nabla \times a) - a \bullet (\nabla \times b)$$
$$\nabla \times (a \times b) = a(\nabla \bullet b) - b(\nabla \bullet a) + (b \bullet \nabla)a - (a \bullet \nabla)b$$

Appendix B Program listing

Fig.B1 Geometry used in the calculations



The Matlab-code (just simple ASCII-files to be read with any ordinary text-editor) is supplied on www.kunstmanen.nl\antennes as two zip-files, one for an RQHA (RQHA_M6.zip) and one for the calculation of a single (horizontal or vertical) and a crossed dipole (DIPOLE_M6.zip).